DISPERSION EFFECTS AND HEAT AND MASS TRANSFER IN FILTRATION FLOWS IN POROUS MEDIA WITH RANDOM INHOMOGENEITIES

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The effect of fluctuations in filtration rates on the coefficients of internal heat and mass transfer and of convective dispersion in randomly inhomogeneous porous media is investigated on the basis of the correlation theory of stationary random functions.

As is well known [1-5], inhomogeneities in the filtration properties of a porous medium (permeability, porosity) lead to the appearance of fluctuations in filtration rates. This effect, called in [3] filtration pseudoturbulence, can lead to various dispersion effects and can have a substantial, sometimes even dominant effect on the various transport processes in filtration flows. A quite large number of studies is known (see [6-11], as well as the review and discussion in [5]), devoted to accounting for these inhomogeneities and investigating the corresponding random fields of velocity, pressure, impurity concentration, and others.

The analysis below of the effect of medium inhomogeneities on heat and mass transfer processes was carried out on the basis of the results of [5, 7], where the correlation theory of stationary random functions [12] is the mathematical tool used.

Filtration in a Granular Layer. The correlation properties of random velocity and pressure fields were investigated in [7] for filtration flow in a densely packed granular layer. With the purpose of removing several inaccuracies in [7], we consider in detail several elements of the theory developed in [7].

The local filtration velocity, fluid pressure, and porosity can be represented in the form

$$\mathbf{V} = \mathbf{v} + \mathbf{v}',\ P = p + p',\ M = m + m', \tag{1}$$

where $\mathbf{v}$, p, and m are the mean velocity, pressure, and porosity, and the primed quantities are the corresponding fluctuations.

The following equations are valid under conditions of validity of the two-term filtration law for an incompressible fluid

$$-\nabla P = (K_1 + K_2 V)\mathbf{V},\ \nabla\mathbf{V} = 0. \tag{2}$$

The coefficients K_1 and K_2 are assumed to be known functions of the porosity m. Substituting (1) into (2) and including only linear terms in the fluctuations leads to stochastic equations, describing fluctuating motion:

$$\begin{gathered} -\nabla p' = (K_1 + K_2 v)\mathbf{v}' + K_2(\mathbf{v}_0\mathbf{v}')\mathbf{v} + (K_1' + K_2' v)\mathbf{v}m', \\ \nabla\mathbf{v}' = 0,\ K_i = K_i(m),\ K_i' = \frac{dK_i}{dm}(m),\ \mathbf{v}_0 = \frac{\mathbf{v}}{v}. \end{gathered} \tag{3}$$

The random fields m', p', $\mathbf{v}'$ are represented in the form of Fourier–Stieltjes integrals with random measures dZ_m, dZ_p, $d\mathbf{Z}_v$. Taking into account relations (3), we then reach linear equations relating these random measures:

$$-i\mathbf{w}dZ_p = (K_1 + K_2 v)\,d\mathbf{Z}_v + K_2(\mathbf{v}_0 d\mathbf{Z}_v)\mathbf{v} + (K_1' + K_2' v)\mathbf{v}\,dZ_m,\ \ \mathbf{w}d\mathbf{Z}_v = 0. \tag{4}$$

The solutions of (4) are:

$$dZ_p = i\alpha(1-\gamma)(K_1 + K_2 v)\frac{w_1}{w^2 - \gamma w_1^2}\,v dZ_m,$$

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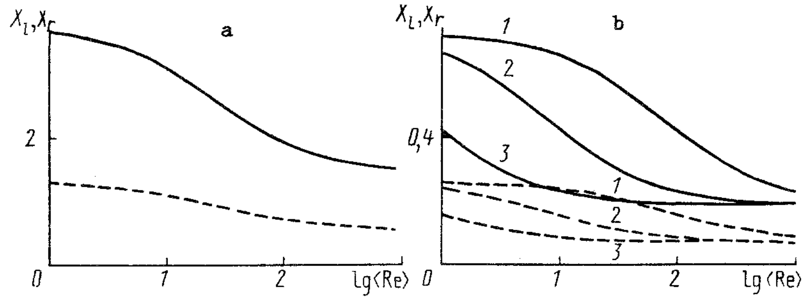

Fig. 1. X_ℓ (solid curve) and X_r (dashed) as functions of ‹Re› for filtration in a granular leyer (a) and filtration in an inhomogeneous porous layer of arbitrary structure (b). Curves 1-3 correspond to the parameter values $Re_* = 10, 1, 0.1$.

$$d\mathbf{Z}_v = \alpha (1-\gamma) \frac{\mathbf{w} w_1 - \mathbf{v}_0 w^2}{w^2 - \gamma w_1^2} v dZ_m, \tag{5}$$

where the following parameters were introduced

$$\alpha = \frac{K_1' + K_2' v}{K_1 + K_2 v}, \quad \gamma = \frac{K_2 v}{K_1 + 2K_2 v}, \tag{6}$$

and the x-axis has been selected along the direction of the mean velocity $\mathbf{v}$. Using for the coefficients K_1 and K_2 the expressions following from the semiempirical Ergun equations [13] for the hydraulic resistance of a granular layer, we find

$$\alpha = \frac{1}{m} \frac{2 - m + 0{,}0233 \langle \mathrm{Re} \rangle}{1 - m + 0{,}0117 \langle \mathrm{Re} \rangle}, \quad \gamma = \frac{0{,}0117 \langle \mathrm{Re} \rangle}{1 - m + 0.0117 \langle \mathrm{Re} \rangle}, \tag{7}$$

$$\langle \mathrm{Re} \rangle = 2av/\nu.$$

In Eqs. (5)-(7) we have removed inaccuracies committed in writing the analogous relations in [7].

The relation found between the random measures $d\mathbf{Z}_v$ and dZ_m makes it possible to calculate in particular, the mean-square components of the velocity fluctuation $\langle v_i'^2 \rangle$ for known spectral properties of m'. Such calculations were carried out in [7], where the following result was obtained

$$\langle v_1'^2 \rangle = X_l^2 v^2 \Delta m, \quad \langle v_2'^2 \rangle = \langle v_3'^2 \rangle = X_r^2 v^2 \Delta m, \tag{8}$$
$$X_l^2 = 0{,}30 [\alpha (1-\gamma)]^2 (J_0 - 2J_2 + J_4),$$
$$X_r^2 = 0{,}15 [\alpha (1-\gamma)]^2 (J_2 - J_4), \quad J_n = \int_{-1}^{1} \frac{t^n dt}{(1-\gamma^2 t^2)^2}.$$

Here Δm is the absolute value of the deviation between the mean layer porosity and some value m_*, for which the porosity inhomogeneities presumably vanish [7].

The dependences of X_ℓ and X_r on ‹Re› are shown in Fig. 1a for m = 0.4.

Filtration in a Porous Medium of Arbitrary Structure. The effect of permeability fluctuations on filtration and impurity transport in a porous medium under conditions of validity of the Darcy law was studied in [5]. For a number of practical calculations, however, it is necessary to know the dependence of the effective heat and mass transfer coefficients on the Reynolds number, including its values when Darcy's law is definitely not satisfied due to the occurrence of inertial effects. To generalize the results of [5] to the large Re region we consider filtration in a medium with random inhomogeneities, replacing Darcy's law by the two-term Dupuit–Forkhgeimer filtration law [14]. The local filtration and continuity equations are also in this case of the form (2), where

$$K_1 = \mu/K, \quad K_2 = \beta\rho/\sqrt{K}. \tag{9}$$

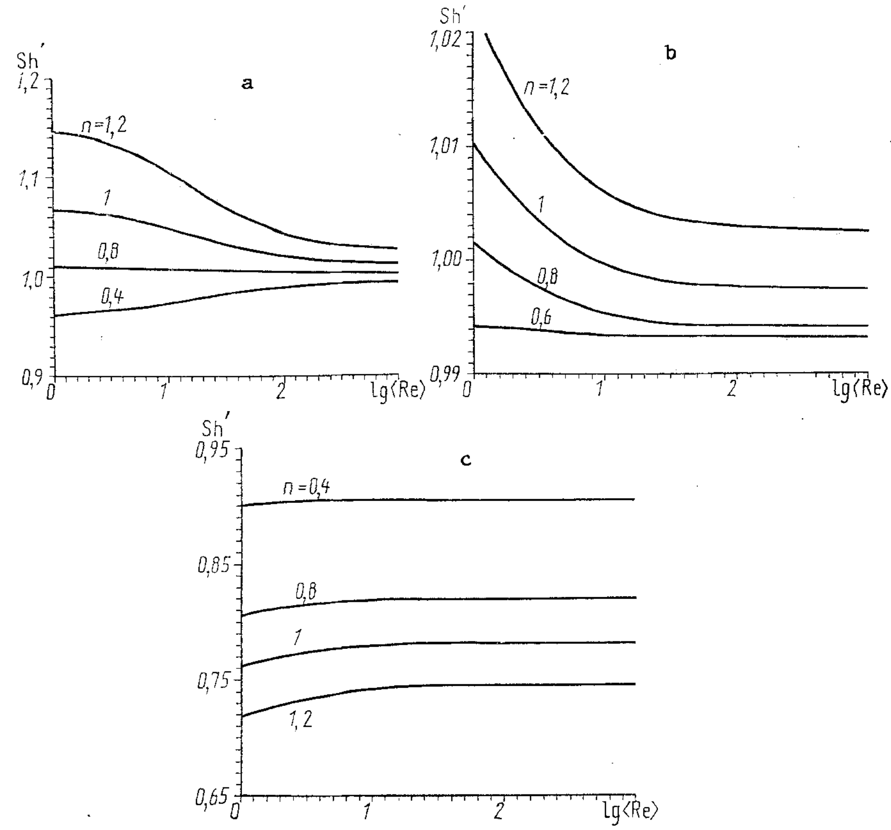

Fig. 2. The mean effective Sherwood number Sh' as a function of <Re> for various n (digits at curves): a) for filtration in a granular layer at $m = 0.4$, $\Delta m = 0.05$; b) for filtration in an inhomogeneous porous medium with $Re_* = 0.1$, $\Delta k = 0.1$, $g = 0$; c) the same in the case of linear dependence of porosity on permeability, i.e., $g = m/k$.

A set of various expressions, obtained on the basis of handling experimental data (see, for example, [1, 4, 15-18]), has been suggested for the dimensionless coefficient β in (9). For simplicity we assume here that β is an empirical quantity, being constant for a given porous medium.

We represent the local velocity, pressure, permeability, and porosity in the form (1). Carrying out calculations similar to (2)-(6), we obtain the same connection between the random measures $d\mathbf{Z}_v$, dZ_p, and dZ_k as in (5), provided dZ_m is replaced by dZ_k. Taking into account (9), the coefficients K_1' and K_2' in (6) are

$$K_1' = -k^{-2}\mu, \quad K_2' = -\frac{1}{2}k^{-3/2}\beta\rho.$$

The last relations determine the parameters α and γ in (6):

$$\alpha = -\frac{1}{k}\frac{1+0.5B\langle \mathrm{Re}\rangle}{1+B\langle \mathrm{Re}\rangle}, \quad \gamma = \frac{B\langle \mathrm{Re}\rangle}{1+2B\langle \mathrm{Re}\rangle}, \qquad (10)$$

$$B = \beta m^{3/2}, \quad \langle \mathrm{Re}\rangle = \frac{k^{1/2}v}{\nu m^{3/2}}.$$

The equation for the Re number suggested by M. D. Millionshchikov was used in (10). Its effectiveness was established experimentally for a large class of porous media [1, 15].

Based on the dependence between the random measures $d\mathbf{Z}_v$ and kZ_k, by carrying out calculations similar to those in [5] we find expressions for the mean-square components of the velocity fluctuations

$$\langle v_1'^2 \rangle = [\alpha(1-\gamma)v]^2 \int \left[\frac{w_1^2 - w^2}{w^2 - \gamma w_1^2}\right]^2 \Phi_{k,k}(w)\, d\mathbf{w} = 0.50\,[\alpha(1-\gamma)v]^2 (J_0 - 2J_2 + J_4)\, k_0^2, \qquad (11)$$

$$\langle v_2'^2 \rangle = \langle v_3'^2 \rangle = 0.25\,[\alpha(1-\gamma)\,v]^2\,(J_2 - J_4)\,k_0^2, \tag{12}$$

where $J_m(\gamma)$ is defined in (8), $\Phi_{k,k}(w)$ is the spectral density of the random permeability field [5], and k_0 has the meaning of mean-square permeability fluctuations

$$k_0^2 = 4\pi \int_0^\infty \Phi_{k,k}(w)\, w^2\, dw.$$

In particular, in satisfying Darcy's law $\gamma = 0$, but $\alpha = -1/k$. In this case we obtain from (11), (12)

$$\langle v_1'^2 \rangle = \frac{8}{15} v^2 \frac{k_0^2}{k^2}, \quad \langle v_2'^2 \rangle = \langle v_3'^2 \rangle = \frac{1}{15} v^2 \frac{k_0^2}{k^2}.$$

The latter relations coincide with the equations obtained in [2] and in [5] for the dispersion of the filtration velocity components under conditions of Darcy's law.

To investigate the nature of dependence of the mean-square components of the velocity fluctuations on <Re> we use the expression for β, selected from the A. I. Charnyi equation [17] for the two-term filtration law. For the parameter B in (10) we have $B = 0.064/Re_*$. The Re_* value in [17] was selected ~ 0.1 from comparison results with experimental data on filtration in granular media.

Relations (11), (12) are conveniently represented in the form (8)

$$\langle v_1'^2 \rangle = X_l^2\, v_l^2\, \Delta k, \quad \langle v_2'^2 \rangle = \langle v_3'^2 \rangle = X_r^2\, v^2 \Delta k,$$
$$X = 0.50\,[k\alpha(1-\gamma)]^2\,(J_0 - 2J_2 + J_4),$$
$$X_r^2 = 0.25\,[k\alpha(1-\gamma)]^2\,(J_2 - J_4). \tag{13}$$

Here $\Delta k = k_0{}^2/k^2$ has the meaning of mean-square relative permeability fluctuations.

The shape of the functions $X_l(\langle Re\rangle)$, $X_r(\langle Re\rangle)$ is shown in Fig. 1b. As follows from the plots, the shape of the plots depends strongly on the parameter Re_*, which may vary strongly for porous media of varying structure.

Internal Heat and Mass Transfer. Numerous empirical equations for the dependence of Sherwood Sh and Nusselt Nu numbers, characterizing the intensity of heat and mass transfer processes to granular layers, on the Re number were obtained in a number of studies (see [4, 10, 19-21]) on the basis of analyzing experimental data on heat and mass transfer to dispersed particles of a granular layer. These equations, having in most cases the shape of power-law dependences, describe very well the results of various experiments, but display poor internal consistency.

One of the reasons for the inconsistency of empirical dependences can be related to the fact that in most experiments there exists an uncontrolled factor, related to the inhomogeneities of the porous medium, leading to generation of filtration of pseudoturbulent flow [3] and affecting the experimentally determined mean coefficients of heat or mass transfer.

To investigate this effect for filtration flow in a granular layer we assume that the approximation $Sh = C\,Re^n$ is valid in some Re interval. Assuming then that the velocity fluctuations are relatively slow, we obtain

$$Re^n = \left(\frac{2a}{\nu}\right)^n V^n,\; V^n = [(v+v_1')^2 + v_2'^2 + v_3'^2]^{n/2} = v^n\left[1 + \frac{2v_1'}{v} + \frac{v'^2}{v^2}\right]^{n/2} = v^n\left[1 + \frac{n}{2}\left(\frac{2v_1'}{v} + \frac{v'}{v^2}\right) + \right.$$
$$\left. + n\left(\frac{n}{2} - 1\right)\frac{v_1'^2}{v^2}\right] + \ldots;\; v'^2 = v_1'^2 + v_2'^2 + v_3'^2.$$

Hence

$$\langle V^n \rangle = v^n\left[1 + \frac{n}{2}(n-1)\frac{\langle v_1'^2 \rangle}{v^2} + \frac{n}{2}\,\frac{\langle v_2'^2 \rangle + \langle v_3'^2 \rangle}{v^2}\right]. \tag{14}$$

Taking now into account Eq. (8), we find

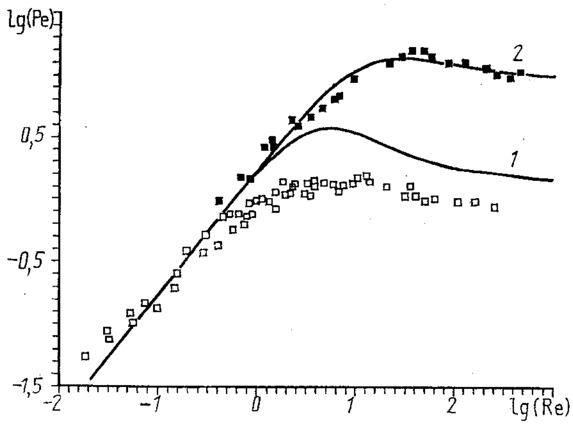

Fig. 3. The quantities Pe_ℓ and Pe_r for a granular layer (curves 1 and 2, respectively) as functions of the Reynolds number. The curves are calculation results by Eqs. (23) at $\beta_\ell = \beta_r = 0.003$, $\psi = 0.45$, $\Delta m = 0.0015$, $C = 1$; the dark and light points correspond to the radial and axial dispersion coefficients, determined experimentally [24].

$$\langle \mathrm{Re}^n \rangle = \langle \mathrm{Re}^n \rangle \left[1 - \left(\frac{n}{2}(1-n)X_l^2 - nX_r^2\right)\Delta m\right],$$

making it possible to write down a relation for the effective mean Sherwood number:

$$\frac{\langle \mathrm{Sh} \rangle}{\mathrm{Sh}(\langle \mathrm{Re} \rangle)} = \mathrm{Sh}' = 1 - \left[\frac{n}{2}(1-n)X_l^2 - nX_r^2\right]\Delta m. \tag{15}$$

The dependences (15) on <Re> are shown in Fig. 2a for various n. Analysis of the figure shows that the effect of porosity fluctuations on the effective mass-exchange coefficient is manifested most strongly for small Reynolds numbers. This effect is attenuated for increasing <Re>, and practically vanishes for <Re> ∿ 1000. The deviation of the effective Sherwood number from the same number, calculated for the mean value of the Reynolds number, can reach 15% (for $\Delta m = 0.05$). The nature of the dependence of Sh' on <Re> is largely determined by the exponent n in the power-law equation for the Sherwood number. As is seen from the figure, for various n the packing inhomogeneity of the granular layer can lead, in principle, to both a decrease (n = 0.4, 0.6) or an increase (n = 0.8, 1.0, 1.2) of the heat or mass transfer coefficient. This conclusion is interesting, since earlier on it has been usually assumed that a nonuniformity in the granular layer can only lead to deterioration of heat or mass transfer in this case [10].

To investigate heat and mass transfer processes during filtration in a porous medium of arbitrary structure with random inhomogeneities we also use a power-law dependence for the Sherwood number. Applying the Newton binomial equation, and assuming that the velocity, permeability, and porosity fluctuations are small, we have

$$\mathrm{Sh} = C\,\mathrm{Re}^n = C\left(\frac{K^{1/2}V}{\nu M^{3/2}}\right)^n = C\left(\frac{k^{1/2}v}{\nu m^{3/2}}\right)^n\left[\left(1+\frac{k'}{k}\right)\left(1-\frac{3m'}{m}\right)\left(1+\frac{2v_1'}{v}+\frac{v'^2}{v^2}\right)\right]^{n/2}.$$

We assume that the porosity M and the permeability K are related deterministically ($M = f(K)$). In that case $m' = gk'$, where $g = df/dK|_{K=k}$. Averaging the latter expression for the Sherwood number, we obtain

$$\mathrm{Sh} = C\langle \mathrm{Re} \rangle^n\left[1 + \left(-\frac{3n^2}{4}\frac{g}{km} + \frac{n}{4}\left(\frac{n}{2}-1\right)k^{-2} + \frac{9n}{4}\left(\frac{n}{2}-1\right)\frac{g^2}{m^2}\right)\langle k'^2 \rangle + \frac{n}{2}(n-1)\frac{\langle v_1'^2 \rangle}{v^2} + \frac{n}{2}\frac{\langle v_2'^2 \rangle + \langle v_3'^2 \rangle}{v^2} + \frac{n^2}{2}\left(\frac{1}{kv} - 3\frac{g}{mv}\right)\langle k' \cdot v_1' \rangle\right]. \tag{16}$$

We note that $\langle k'^2 \rangle = k_0^2$. Carrying out calculations similar to those used in deriving (11), (12), we find

$$\langle k'v_1' \rangle = \frac{vk_0^2}{k} X_{lk}, \quad X_{lk} = 0{,}5k\alpha(1-\gamma)\int_{-1}^{1}\frac{x^2-1}{1-\gamma x^2}dx. \tag{17}$$

Expressions (15), (17) make it possible to write Eq. (16) in the following form:

$$\mathrm{Sh}' = \frac{\langle \mathrm{Sh} \rangle}{\mathrm{Sh}(\langle \mathrm{Re} \rangle)} = 1 + \left\{\left[\frac{n}{4}\left(\frac{n}{2}-1\right) - \frac{3n^2}{4}\frac{gk}{m} + \frac{9n}{4}\left(\frac{n}{2}-1\right)\frac{g^2k^2}{m^2}\right] + \frac{n}{2}(n-1)X_l^2 + nX_r^2 + \frac{n^2}{2}\left(1-3\frac{gk}{m}\right)X_{lk}\right\}\Delta k. \tag{18}$$

Figure 2b shows the dependence of Sh' on <Re> for various values of n and of the parameter Re_* for situations in which the medium porosity does not undergo fluctuations (i.e., is constant). Figure 2c shows the same dependences for the case of linear deterministic dependence of the porosity on permeability (g = m/k). It is noted that the curve shapes depend strongly on both n and on the parameters Re_* and gk/m, which can acquire values differing by orders of magnitude. A large effect on the shape of the dependence Sh'(<Re>) is, obviously, also due to the choice of expressions for the Re number (12) in the empirical power-law equation for the Sherwood number.

Convective Dispersion. During flow in porous and granular media, along with molecular diffusion and convective transport there exists one more impurity transport mechanism, called convective dispersion. This effect is generated by the displacements of neighboring fluid jets during motion in the intersecting pore space. The presence of a distinct flow direction along the instrument axis, along which there occurs filtration transport, also leads to the fact that the total coefficient of impurity dispersion D acquires a tensor character, where the axial component of the dispersion coefficient D_ℓ exceeds substantially the radial component D_r. Besides, nonmonotonic dependences of the Peclet number Pe = 2av/D on the Reynolds number were observed experimentally [22], for which the appearance of local maxima on the curve Pe(Re) is characteristic. In a number of studies (see, for example, [23, 24]) the authors succeeded in describing the experimentally observed effects, but in their constructions they did not succeed in avoiding the use of phenomenological hypotheses and dependences. It is shown below that all observed effects can be explained as a result of inhomogeneities of the porous medium.

For filtration transport of neutral impurities in a homogeneous porous medium the effective dispersion coefficients D_ℓ and D_r can be represented in the following form (if the impurity concentration is defined for unit volume of the porous medium) [7, 11]:

$$D_l = \psi D_m + 2a\beta_l v, \quad D_r = \psi D_m + 2a\beta_r v, \tag{19}$$

where ψ is the product of the twisting coefficient by the porosity, D_m is the molecular impurity diffusion coefficient in the fluid, and β is an empirical coefficient.

A quite rigorous analysis of the effect of random inhomogeneities on impurity dispersion in a porous medium can be carried out on the basis of averaging the equations of convective diffusion, as was done in [5] for filtration under conditions of the Darcy law. Due to the unwieldiness of this analysis we restrict ourselves to a simplified study, based on the results obtained in [7] for filtration in a densely packed granular layer.

We express the axial D_ℓ' and radial D_r' dispersion coefficients due to inhomogeneities of the porous medium in the form

$$D_l' = \langle v_l'^2 \rangle T, \quad D_r' = \langle v_2'^2 \rangle T, \tag{20}$$

where T is the mean lifetime of fluctuations in the impurity concentration within the limits of the sample. This time can be represented, in turn, in the form

$$T = 2L^2/(D_l + D_r), \tag{21}$$

where L is the spatial scalar scale of local fluid motions. We put $L^2 = 4a^2C$, where C is a numerical dimensionless parameter. For flow in the granular layer C ∿ 1, since the characteristic inhomogeneity scale of a granular layer is comparable with the layer particle sizes. For filtration flows in porous media of a different structure (cemented rock) this parameter can acquire large values, since for a substantial class of porous media the inhomogeneity scale exceeds strongly the size of an isolated pore.

From (19)-(21) we have

$$\langle D_l \rangle = D_l + \frac{8Ca^2 \langle v_1'^2 \rangle}{D_l + D_r}, \quad \langle D_r \rangle = D_r + \frac{8Ca^2 \langle v_2'^2 \rangle}{D_l + D_r}. \tag{22}$$

Introducing the Schmidt $Sc = \nu/D_m$ and Peclet $Pe_\ell = 2av/\langle D_\ell \rangle$, $Pe_r = 2av/\langle D_r \rangle$ ($Re = 2av/\nu$) numbers, we rewrite relation (22) in dimensionless form

$$\frac{1}{Pe_l} = \frac{\psi}{Sc\,Re} + \beta_l + \frac{2C\langle v_1'^2 \rangle / v^2}{\frac{2\psi}{Sc\,Re} + \beta_l + \beta_r},$$
$$\frac{1}{Pe_r} = \frac{\psi}{Sc\,Re} + \beta_r + \frac{2C\langle v_2'^2 \rangle / v^2}{\frac{2\psi}{Sc\,Re} + \beta_l + \beta_r}. \tag{23}$$

These dependences (as functions of the Reynolds number) are presented in Fig. 3 for filtration in a granular layer, when the quantities $\langle v_i'^2 \rangle / v^2$ are calculated by Eqs. (8). Also provided here are experimental data [24] for varying radial and axial dispersion. Similar dependences for impurity filtration transport in an inhomogeneous porous medium are easily obtained by using values for the mean-square components of velocity fluctuations from relations (11), (12).

NOTATION

a, particle radius of a granular layer; B, parameter in (10); C, a proportionality constant; D, dispersion coefficient; D_m, molecular diffusion coefficient; dZ_m, dZ_k, dZ_p, $d\mathbf{Z}_v$, random measures; J_n, auxiliary function in (8); K, M, P, V, and k, m, p, v are, respectively, the local and mean permeability, porosity, pressure, and velocity; K_1 and K_2, coefficients in (2); L, scale of local fluid motion; n, parameter in (15); Pe, Peclet number; Re, Reynolds number; Re_*, a parameter characterizing the filtration properties of a porous medium; Sc, Schmidt number; Sh, Sherwood number; Sh', mean effective Sherwood number; T, some characteristic time in (20); v_0, unit vector defined in (3); $\mathbf{w}$, wave vector; X_ℓ, X_r, $X_{\ell,k}$, functions of Re defined in (8), (13), (17), α, γ, are coefficients in (5); β, coefficient in (9); μ, viscosity; ν, kinematic viscosity; ρ, density; $\Phi_{k,k}$, spectral density; ψ, porosity-twisting product; and $\langle\ \rangle$, averaging operator. Subscripts ℓ and r are the radial and axial components of a quantity, and ' denotes random fields with vanishing means.

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EQUATIONS OF HYDRODYNAMICS FOR POROUS MEDIA WITH A VOID STRUCTURE POSSESSING FRACTAL GEOMETRY

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We obtain the equations of filtration for the case when a void space is a fractal embedded in a continuous medium. We consider a model of capillary permeation of porous materials with percolation properties.

Introduction. In recent years the theory of fractals, i.e., objects with a fractional spatial dimensionality [1-4], has been widely used to describe the structure of disordered media and processes in disordered media. Examples of disordered materials are porous bodies such as rocks. The void space, the skeleton, or the surface of the body can be a fractal [5, 6].

It was shown in [7, 8] that certain features of the behavior of processes in porous media are determined by the percolation properties of the void space of rocks. It is known (see [9], for example) that a percolation cluster has fractal properties. Hence it follows that the void space of sedimentary rocks has fractal properties, which is observed experimentally [10].

1. Multiphase Filtration. We first consider the equations of multiphase filtration for the case when one phase (the wetting phase, for example) extrudes the other (nonwetting) phase.

We assume that the void space is a fractal with the Hausdorff–Besikovitch dimensionality d_f embedded in a continuous medium with dimensionality d ($d \geq d_f$, d = 2, 3).

To derive the equations we adopt the method used in [11] to derive the equations of filtration in a fractured medium with the cracks having a fractal geometry.

We consider flow with cylindrical (d = 2) or spherical (d = 3) symmetry, when all functions depend only on the time t and the distance r from the center of symmetry. Then the integral conservation of mass equation is written in the form

$$\frac{\partial}{\partial t}\int\limits_{r_0\leqslant r\leqslant r_1} m\rho_w S_w d\overset{d_f}{\mu_p}=\int\limits_{r=r_0} q_f d\overset{d_f}{\mu_s}-\int\limits_{r=r_1} q_f d\overset{d_f}{\mu_s}, \tag{1}$$

where $d\overset{d_f}{\mu_p} = dr d\overset{d_f}{\mu_s}$.

Below we will need the relation

$$\int\limits_{r=r_1} d\overset{d_f}{\mu_s}=r_1^{d_f-1}\int\limits_{r=1} d\overset{d_f}{\mu_s}=r_1^{d_f-1}\alpha_{d_f}, \tag{2}$$

where $\alpha_{d_f} = 2\pi^{d_f/2}\Gamma^{-1}(d_f/2)$ is the surface area of a unit $(d_f - 1)$-dimensional sphere.

Using (2), we rewrite (1) in the form

$$m\rho_w\frac{\partial S_w}{\partial t}=\frac{1}{r^{d_f-1}}\;\frac{\partial}{\partial r}\,[r^{d_f-1}q]. \tag{3}$$

Combining (3) with the equation of the generalized Darcy's law

$$\mathbf{u}_i=-\frac{k}{\mu_i}f_i(S_w)\nabla p_i, \tag{4}$$

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